AP Calculus AB/BC Summer Packet
Welcome!

This packet includes a sampling of problems that students entering AP Calculus should be able to answer without much hesitation. The questions are organized by topic:

Section	Topic	AB, do…	BC, do…
A	Algebra Skills	all	all
T	Trigonometry	all	all
L	Logarithmic and Exponential Functions	all	all
F	Factoring (Including Higher-Order Factoring)	part	all
R	Rational Expressions and Equations	part	all
I	Interpreting Graphs	part	all
G	Graphing	all	all
C	Calculator Skills	all	all
S	Series (BC only)	none	all
P	Parametric and Polar Equations (BC only)	none	all

Students planning on taking **AP Calc BC** should do **ALL** the problems in this packet.
Students planning on taking **AP Calc AB** may skip the problems labeled "BC only."

Questions should be answered **WITHOUT A CALCULATOR** unless otherwise stated.
Throughout the packet—and throughout Calculus—angles are in *radians*, not degrees.

Students entering AP Calculus *must* have a strong foundation in algebra as well as trigonometry. Most questions in this packet concern skills and concepts that will be used extensively in AP Calculus. Others have been included not so much because they are skills that are used frequently, but because being able to answer them indicates a strong grasp of important mathematical concepts and—more importantly—the ability to problem-solve.

An answer key is provided at the end of this file. It is extremely important for students to review the concepts contained in this packet. Students who choose not to complete the packet may:

a) lack the prerequisite skills necessary for success in AP Calculus, or
b) lack the work ethic necessary for success in AP Calculus.

This course requires students to represent problems in multiple ways (numerically, algebraically, graphically), and to justify answers using clear, logical analysis. **This is not a course where every problem given on a test or quiz is identical to problem types done repeatedly during class. Tests and quizzes—as well as the AP exam—require students to apply skills and concepts to seemingly new situations and/or to connect multiple mathematical ideas.**

Take a deep breath and begin working—pace yourself; don't try to complete the packet in one sitting at the last minute. If you have "the basics" down and you put in the necessary work, you will see how amazing Calculus is and how it is the capstone to all mathematics you have learned thus far!

A

Fundamental Algebra Skills

★ ★ This entire page is for AB *and* BC students. ★ ★

NO CALCULATOR

A1. True or false. If false, change what is underlined to make the statement true.

a. $\left(x^3\right)^4 = x^{\underline{12}}$ T F

b. $x^{\frac{1}{2}}x^3 = x^{\underline{\frac{3}{2}}}$ T F

c. $(x + 3)^2 = \underline{x^2 + 9}$ T F

d. $\dfrac{x^2 - 1}{x - 1} = \underline{x}$ T F

e. $(4x + 12)^2 = \underline{16}(x + 3)^2$ T F

f. $\underline{3} + 2\sqrt{x - 3} = 5\sqrt{x - 3}$ T F

A2. More algebra.

a. If 6 is a zero of *f*, then ____ is a solution of $f(2x) = 0$.

b. Lucy has the equation $2(4x + 6)^2 - 8 = 16$. She multiplies both sides by ½. If she does this correctly, what is the resulting equation?

c. Simplify $\dfrac{2 \pm 4\sqrt{10}}{2}$

d. Rationalize the denominator of $\dfrac{12}{3 + \sqrt{x - 1}}$

e. If $f(x) = 3x^2 + x + 5$, then $f(x + h) - f(x) =$ (Give your answer in simplest form.)

f. A cone's volume is given by $V = \frac{1}{3}\pi r^2 h$. If $r = 3h$, write V in terms of h only.

g. An equilateral triangle has side length *s*. Write an expression for its area in terms of *s*.

T

Trigonometry

★ ★ This entire page is for AB *and* BC students. ★ ★

NO CALCULATOR

You should be able to answer T1 and T2 quickly, *without* referring to (or drawing) a unit circle.

T1. Find the value of each expression, in exact form.

a. $\sin \frac{2\pi}{3}$

b. $\cos \frac{11\pi}{6}$

c. $\tan \frac{3\pi}{4}$

d. $\arctan(\sqrt{3})$

e. $\sec^2\left(\frac{5\pi}{6}\right)$

f. $\arccos\left(-\frac{\sqrt{2}}{2}\right)$

T2. Find the value(s) of x in $[0, 2\pi)$ which solve each equation.

a. $\sin x = \frac{\sqrt{3}}{2}$

b. $\cos x = -1$

c. $\sec x = -2$

d. $\tan x = \sqrt{3}$

e. Explain how question **(d)** in **T2** is different from question **(d)** in **T1.**

f. $1 + 2\sin(3x) = 0$

g. $\cot(2x) = 1$

T3. Solve by factoring. Give solution(s) in $[0, 2\pi)$ only

a. $4\sin^2 x + 4\sin x + 1 = 0$

b. $\cos^2 x - \cos x = 0$

c. $\sin x \cos x - \sin^2 x = 0$

d. $x \tan x - 3\tan x = x - 3$

T4. Verify the following trigonometric identities.

Note: You are expected to know the three Pythagorean trigonometric identities and two double-angle identities—sin(2x) and cos(2x).

a. $\tan x + \cot x = (\sec x)(\csc x)$

b. $\dfrac{\cos x}{1 - \sin x} = \dfrac{1 + \sin x}{\cos x}$

c. $(\sin x + \cos x)^2 = 1 + \sin(2x)$

L

Logarithmic and Exponential Functions

★ ★ This entire page is for AB *and* BC students. ★ ★

NO CALCULATOR

L1. Expand as much as possible.

a. $\ln x^2y^3$

b. $\ln \dfrac{x+3}{4y}$

c. $\ln 3\sqrt{x}$

d. $\ln 4xy$

L2. Condense into the logarithm of a single expression.

a. $4\ln x + 5\ln y$

b. $\frac{2}{3}\ln a + 5\ln 2$

c. $\ln x - \ln 2$

d. $\dfrac{\ln x}{\ln 2}$ (contrast with part **c**)

L3. Solve. Give your answer in exact form.

a. $\ln (x + 3) = 2$

b. $\ln x + \ln 4 = 1$

c. $e^{4x+5} = 1$

d. $2^x = 8^{4x-1}$

e. $e^{3\ln x} = 64$

f. $\ln x + \ln (x + 2) = \ln 3$

L4. Consider the function given by $f(x) = a(e^{bx})$ where a and b are constants.

a. Find the value of constants a and b so that $f(0) = 2$ and $f(1) = 10$.

b. Rewrite the function in the form $f(x) = c(d)^x$ where c and d are constants.

L5. **YOU MAY USE A CALCULATOR ON THIS QUESTION ONLY.**
At $t = 0$ there were 140 million bacteria cells in a petri dish. After 6 hours, there were 320 million cells. Suppose the population grew exponentially for $t \geq 0$.

a. To the nearest million, how many cells were in the petri dish 11 hours after the experiment began?

b. After how many hours will there be 1 billion cells? Round to the nearest tenth of an hour.

F — Higher-Order Factoring

NO CALCULATOR

F1. Solve by factoring.

a. $x^3 + 5x^2 - x - 5 = 0$

b. $4x^4 + 36 = 40x^2$

c. $(x^3 - 6)^2 + 3(x^3 - 6) - 10 = 0$

d. $x^5 + 8 = x^3 + 8x^2$

F2. Solve by factoring. You should be able to solve each of these *without* multiplying the whole thing out. (In fact, for goodness' sake, please *don't* multiply it all out!)

a. $(x + 2)^2 (x + 6)^3 + (x + 2)(x + 6)^4 = 0$

b. $(2x - 3)^3 (x^2 - 9)^2 + (2x - 3)^5 (x^2 - 9) = 0$

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c. $(3x + 11)^5 (x + 5)^2 (2x - 1)^3 + (3x + 11)^4 (x + 5)^4 (2x - 1)^3 = 0$

d. $6x^2 - 5x - 4 = (2x + 1)^2 (3x - 4)$

F3. Solve. Each question *can* be solved by factoring, but there are other methods, too.

a. $a(3a + 2)^{1/2} + 2(3a + 2)^{3/2} = 0$

b. $\sqrt{2x^2 - 7x + 6} - \sqrt{2x - 3} = 0$

c. $2\sqrt{x + 3} = x + 3$

d. $\dfrac{6}{(2x + 1)^2} + \dfrac{3}{2x + 1} = 1 + \dfrac{2}{2x + 1}$

R	**Rational Expressions and Equations**
	NO CALCULATOR

R1.

	Function	Domain	Hole(s): (x, y) if any	Horiz. Asym., if any	Vert. Asym.(s), if any
a.	$f(x)=\dfrac{4x^2+7x-15}{8x^2-14x+5}$				
b.	$f(x)=\dfrac{3(4+x)^2-48}{x}$				
c.	$f(x)=\dfrac{6x+4}{\sqrt{3x^2-10x-8}}$		skip	skip	

R2. Find the *x*-coordinates where the function's output is zero and where it is undefined.

a. For what real value(s) of *x*, if any, is the output of $f(x)=\dfrac{x^2-4}{x^2-4x+4}$ …

…equal to zero? …undefined?

b. For what real value(s) of *x*, if any, is the output of $\dfrac{x^2+2x}{x^2+5x+6}$ …

…equal to zero? …undefined?

R3. Simplify completely.

a. $\dfrac{2}{\sqrt{x^2+4}}-\dfrac{x^2+4}{3}$ (Don't worry about rationalizing)

b. $\dfrac{3}{\left(\frac{4}{x}\right)^2+1}$ (Your final answer should have just one numerator and one denominator)

c. $\dfrac{5}{x^2+3x+2}-\dfrac{2x}{x^2+2x+1}$

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d. $\dfrac{3}{(x+2)^{1/2}}+\dfrac{x}{(x+2)^{5/2}}$ (Don't worry about rationalizing)

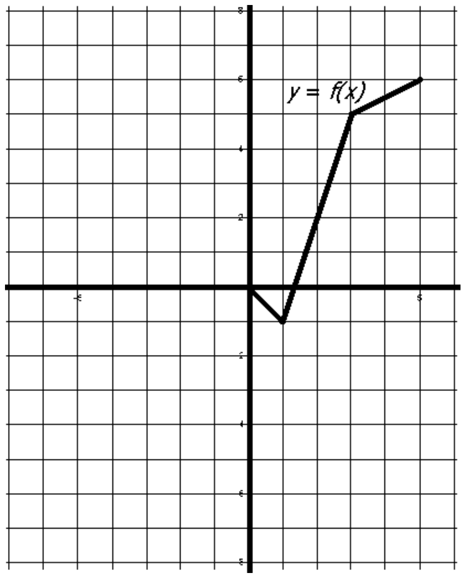
R4. *(Answers may vary)* Write the equation of a function…

a. ….that has asymptotes $y = 4$ and $x = 1$, and a hole at (3, 5)

b. …that has holes at (-2, 1) and (2, -1), an asymptote at $x = 0$, and no horizontal asymptote

I	**<u>Interpreting Graphs</u>** NO CALCULATOR

I1. <u>PART</u> of the graph of f is given. Each gridline represents 1 unit.



a. Complete the graph to make f an EVEN function.

b. What are the domain and range of f_{even}?

c. What is $f_{\text{even}}(-3)$?

d. Complete the graph to make f an ODD function.

e. What are the domain and range of f_{odd}?

f. What is $f_{\text{odd}}(-3)$?

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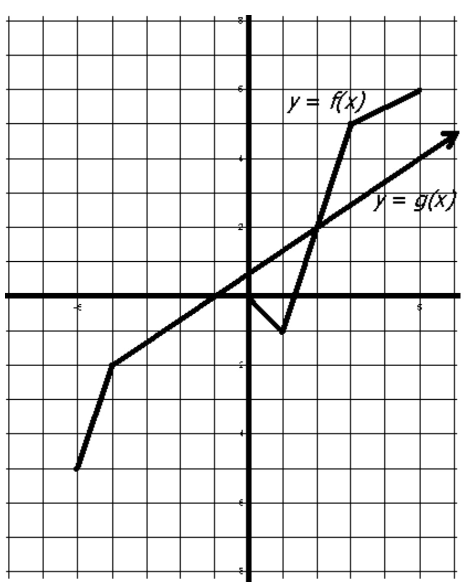
I2. The graphs of f and g are given. Answer each question, if possible. If impossible, explain why. Each gridline represents 1 unit. The domain of f is $[0, 5]$, and the domain of g is $[-5, \infty)$



a. $f^{-1}(5) =$

b. $f(g(5)) =$

c. $(g \circ f)(3) =$
Note: $(g \circ f)(3)$ means the same as $g(f(3))$

d. Solve for x: $f(g(x)) = 5$

e. Let $h(x) = f(x) - g(x)$.
Sketch a graph of h, with domain $[0, 5]$.

f. Let $j(x) = |g(x)|$.
Sketch a graph of j, with domain $[-5, 0]$.

G

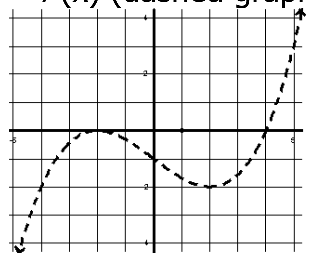
Graphing

★ ★ This entire page is for AB *and* BC students. ★ ★

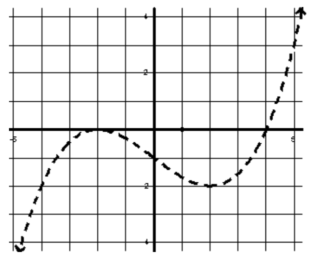
NO CALCULATOR

G1. Given the graph of $y = f(x)$ (dashed graph), sketch each transformed graph.

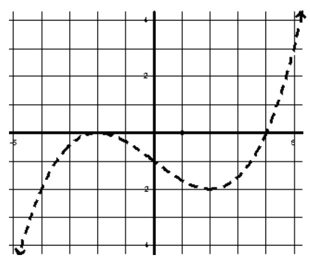
a. $y = f(x + 2)$



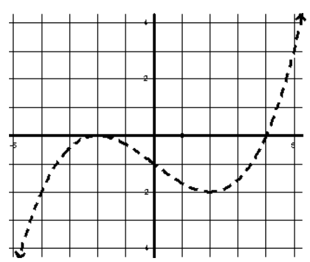
b. $y = 2f(x)$



c. $y = |f(x)|$



d. $y = f(2x) + 1$



G2. Sketch each graph each equation *without* making a table of values. Your graph does not have to be perfect, but it should have the correct shape and should clearly show major characteristics (intercepts, asymptotes, vertex, etc.) Remember, NO CALCULATOR.

a. $y = 2|x + 1|$

b. $y = -\ln(x + 3)$

c. $y = 1 + \cos(x)$

d. $y = 4 - (x + 3)^2$

e. $y = 1 + e^{-x}$

f. $y = \sqrt{x - 4}$

g. $y = \left|\frac{1}{x}\right|$

h. $y = \begin{cases} 2x + 1 & x < 1 \\ 3 - x & x \geq 1 \end{cases}$

C

Calculator Skills

★ ★ This entire section is for AB *and* BC students. ★ ★

NO CALCULATOR

C1. Use a graphing calculator to answer each question, accurate to three decimal places. NO WORK NEEDS TO BE WRITTEN DOWN. Let your calculator do the work!

a. Find the zero(s) of $f(x) = x^5 + 7x^2 - 4$

b. Find the zero(s) of $g(x) = x - \cos x$

c. Find the solution(s) of $x^3 = \cos x$

d. Find the solution(s) of $e^{-x} = \sin x$ on the interval [0, 5].

e. Use your calculator to solve this system of equations graphically:

$$\begin{cases} x^2 + y^2 = 25 \\ x + y = 6 \end{cases}$$

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Students entering AP Calculus AB:
You have reached the end of your packet.

Students entering AP Calculus BC:
You should do the next page as well.

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S	**Series** ★ ★ This entire section is for BC students only. ★ ★ NO CALCULATOR

S1. Write an explicit (*not recursive*) rule for the pattern in each sequence

a.

n	1	2	3	4	5
a_n	10	14	18	22	26

b.

n	1	2	3	4	5
a_n	3	6	12	24	48

c.

n	1	2	3	4	5
a_n	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{24}$	$\frac{1}{120}$

d.

n	1	2	3	4	5
a_n	$\frac{2}{1}$	$\frac{3}{4}$	$\frac{4}{9}$	$\frac{5}{16}$	$\frac{6}{25}$

S2. Find the sum.

a. $\sum_{n=3}^{6} n^2$

b. $\sum_{n=1}^{3} \frac{n}{n+1}$

P	**Parametric and Polar Equations** ★ ★ This entire section is for BC students only. ★ ★ NO CALCULATOR

P1. Convert to rectangular (Cartesian) form, then graph, indicating the orientation of the curve. Remember, you should be doing this *without* a calculator.

a. $(x, y) = (-t, t^2)$ $\quad t \geq 1$

b. $(x, y) = (\sin t, \cos t)$, $0 \leq t \leq 2\pi$

c. $(x, y) = (1 + 4t, \ 3 - t)$
No restrictions on t

d. $(x, y) = (5, t)$
No restrictions on t

P2. Convert between rectangular (Cartesian) and polar coordinates.
Remember, you should be doing this *without* a calculator.

a. $(r, \theta) = (10, \frac{7\pi}{6}) \rightarrow (x, y) = ?$

b. $(x, y) = (8, -8) \rightarrow (r, \theta) = ?$
Use $r > 0$ and $0 \leq \theta < 2\pi$

P3. Convert to rectangular (Cartesian) form, then graph.
Remember, you should be doing this *without* a calculator.

a. $r = 3$

b. $\theta = \frac{\pi}{3}$

c. $r \sin\theta = 2$

d. $r = 5\sec\theta$

★ ★ ★ ANSWER KEY ★ ★ ★

A1.

a. true

b. false; 7/2

c. false; $x^2 + 6x + 9$

d. false; $x + 1$

e. true

f. false; $3\sqrt{x-3}$

A2.

a. $x = 3$

b. $(4x + 6)^2 - 4 = 8$

c. $1 \pm 2\sqrt{10}$

d. $\dfrac{36 - 12\sqrt{x-1}}{10 - x}$ or $\dfrac{12(3 - \sqrt{x-1})}{10 - x}$

e. $6xh + h + 3h^2$

f. $3\pi h^3$

g. $\dfrac{\sqrt{3}}{4}s^2$

T1.

a. $\frac{\sqrt{3}}{2}$

b. $\frac{\sqrt{3}}{2}$

c. -1

d. $\frac{\pi}{3}$

e. $\frac{4}{3}$

f. $\frac{3\pi}{4}$

T2.

a. $\frac{\pi}{3}$, $\frac{2\pi}{3}$

b. π

c. $\frac{2\pi}{3}$, $\frac{4\pi}{3}$

d. $\frac{\pi}{3}$, $\frac{4\pi}{3}$

e. The result of arctangent (**T1d**) is restricted to the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, but the result for **T2d** is restricted to the interval $[0, 2\pi)$.

f. $\frac{7\pi}{18}$, $\frac{11\pi}{18}$, $\frac{19\pi}{18}$, $\frac{23\pi}{18}$, $\frac{31\pi}{18}$, $\frac{35\pi}{18}$

g. $\frac{\pi}{8}$, $\frac{5\pi}{8}$, $\frac{9\pi}{8}$, $\frac{13\pi}{8}$

T3.

a. $\frac{7\pi}{6}$, $\frac{11\pi}{6}$

b. 0, $\frac{\pi}{2}$, $\frac{3\pi}{2}$

c. 0, $\frac{\pi}{4}$, π, $\frac{5\pi}{4}$

d. $\frac{\pi}{4}$, $\frac{5\pi}{4}$, 3

T4.

a.

$$\tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$
$$= \frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^2 x}{\sin x \cos x}$$
$$= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$
$$= \frac{1}{\sin x \cos x}$$
$$= \left(\frac{1}{\cos x}\right)\left(\frac{1}{\sin x}\right)$$
$$= (\sec x)(\csc x)$$

b.

$$\frac{\cos x}{1 - \sin x} = \frac{(\cos x)(1 + \sin x)}{(1 - \sin x)(1 + \sin x)}$$
$$= \frac{(\cos x)(1 + \sin x)}{1 - \sin^2 x}$$
$$= \frac{(\cos x)(1 + \sin x)}{\cos^2 x}$$
$$= \frac{1 + \sin x}{\cos x}$$

c.

$$(\sin x + \cos x)^2 = \sin^2 x + 2\sin x \cos x + \cos^2 x$$
$$= (\sin^2 x + \cos^2 x) + (2\sin x \cos x)$$
$$= 1 + \sin(2x)$$

L1. **a.** $2\ln x + 3\ln y$ **b.** $\ln(x+3) - \ln 4 - \ln y$

c. $\ln 3 + \frac{1}{2}\ln x$ **d.** $\ln 4 + \ln x + \ln y$

L2. **a.** $\ln(x^4y^5)$ **b.** $\ln(32a^{2/3})$

c. $\ln\left(\frac{x}{2}\right)$ **d.** $\log_2 x$ (Change of base vs. log quotient rule)

L3. **a.** $x = e^2 - 3$ **b.** $x = \frac{e}{4}$

c. $x = -\frac{5}{4}$ **d.** $x = \frac{3}{11}$

e. $x = 4$ **f.** $x = 1$ only ($x = -3$ is extraneous)

L4. **a.** $a = 2$, $b = \ln 5$

b. $f(x) = 2(5)^x$

L5. **a.** 637 million

b. 14.3 hours

F1. **a.** $x = -5, -1, 1$

b. $x = -3, -1, 1, 3$

c. $x = 1, 2$

d. $x = -1, 1, 2$

F2. **a.** $x = -6, -4, -2$

b. $x = -3, 0, \frac{3}{2}, \frac{12}{5}, 3$

c. $x = -9, -5, -4, -\frac{11}{3}, \frac{1}{2}$

d. $x = -\frac{1}{2}, 0, \frac{4}{3}$

F3. **a.** $a = -\frac{2}{3}, -\frac{4}{7}$

b. $x = \frac{3}{2}, 3$

c. $x = -3, 1$

d. $x = -\frac{3}{2}, 1$

R1.	**a.**	$x \neq \frac{1}{2}, x \neq \frac{5}{4}$	$\left(\frac{5}{4}, \frac{17}{6}\right)$	$y = \frac{1}{2}$	$x = \frac{1}{2}$
	b.	$x \neq 0$	(0, 24)	none	none
	c.	$\left(-\infty, -\frac{2}{3}\right) \cup (4, \infty)$	skip	skip	$x = 4$

R2. **a.** equal to zero at $x = -2$ undefined at $x = 2$

b. equal to zero at $x = 0$ undefined at $x = -3$ and $x = -2$

R3. **a.** $\dfrac{6-(x^2+4)^{3/2}}{3\sqrt{x^2+4}}$

b. $\dfrac{3x^2}{16+x^2}$

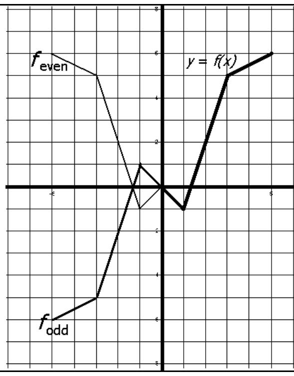
c. $\dfrac{-2x^2+x+5}{(x+1)^2(x+2)}$

d. $\dfrac{3x^2+13x+12}{(x+2)^{5/2}}$ or $\dfrac{(3x+4)(x+3)}{(x+2)^{5/2}}$

R4. **a.** Answers vary. One possibility: $\dfrac{2(x-3)(2x-1)}{(x-3)(x-1)}$

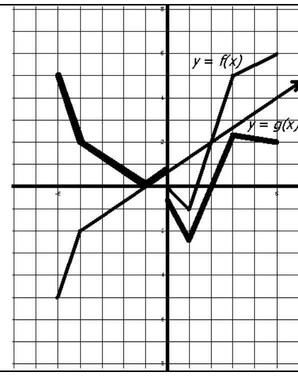
b. Answers vary. One possibility: $\dfrac{(x+2)(x-2)(x^2-6)}{x(x+2)(x-2)}$

I1.



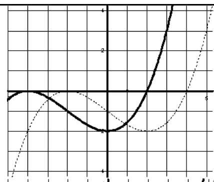
a. see graph
b. D: [-5, 5]
R: [-1, 6]
c. 5
d. see graph
e. D: [-5, 5]
R: [-6, 6]
f. -5

I2.

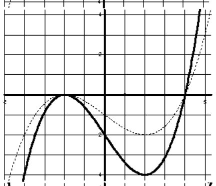


a. 3
b. 5.5
c. 4
d. $x = 3.5$
e. see graph
f. see graph

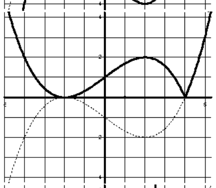
G1. **a.**



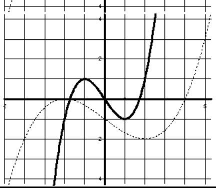
b.



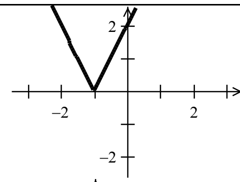
c.



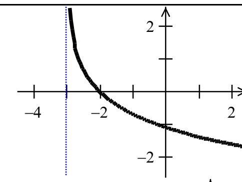
d.



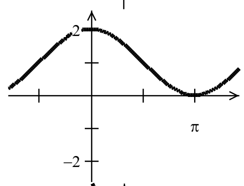
G2. **a.**



b.



c.



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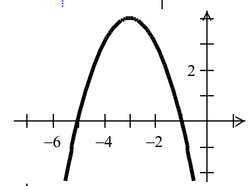
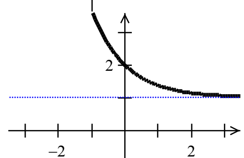


e.



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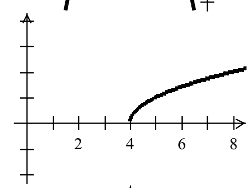
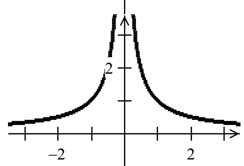


g.



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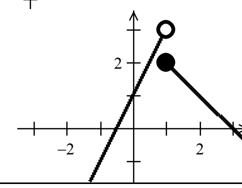
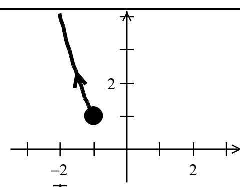


C1. **a.** -1.792, -0.783, 0.735
b. 0.739
c. $x \approx 0.865$
d. $x \approx 0.589, 3.096$
e. $(x, y) \approx (1.129, 4.871)$
$(x, y) \approx (4.871, 1.129)$

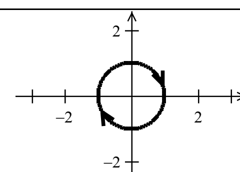
S1. **a.** $a_n = 4n + 6$ **b.** $a_n = 3(2)^{n-1}$ or $1.5(2)^n$
c. $a_n = \frac{1}{n!}$ **d.** $a_n = \frac{n+1}{n^2}$

S2. **a.** 86 **b.** $\frac{23}{12}$

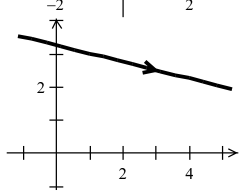
P1. **a.** $y = x^2$



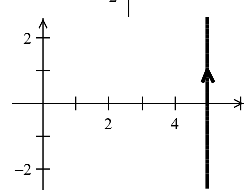
b. $x^2 + y^2 = 1$



c. $y = -\frac{1}{4}x + \frac{13}{4}$



d. $x = 5$



P2. **a.** $(x, y) = (-5\sqrt{3}, -5)$ **b.** $(r, \theta) = (8\sqrt{2}, \frac{7\pi}{4})$

P3. **a.** $x^2 + y^2 = 9$ **b.** $y = (\sqrt{3})x$
c. $y = 2$ **d.** $x = 5$